**Appendix G**
**Section III: Estimation of Joint Probabilities**

This section provides a justification for using the following formula to estimate the probability that zero cattle (P(X=0)) will be imported into the U.S. using the joint probabilities from Branch Points 2 and 3.

$$P(X=0) = (1-r)^m$$

Table 13 illustrates a method of estimating joint probabilities with simple multiplication of individual probabilities for Branch Points 2 and 3 using a fictitious simple example of 10 cattle imported to the United States during the year with $p_2$ = .05 (J220) and $p_3$ = .0001 (J221).

**Table 13. Joint Probabilities from Branch Points 2 and 3**

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 219 | Entering | | Entering | | Entering | | | Cattle imported/year: | | 10 |
| 220 | Quarantine | | Quarantine | | U.S. Port | | | Probability p2: | | 0.05 |
| 221 | Number | | Probability | | Probability | | | Probability p3: | | 0.0001 |
| 222 | Viremic | | k Viremic | | 0 Viremic | | Joint | | | |
| 223 | Cattle (k) | | Cattle | | Cattle | | Probability | | | |
| 224 | | | | | | | | | | |
| 225 | 0 | | 0.598737 | | 1 | | 0.598737 | | | |
| 226 | 1 | | 0.315125 | | 0.9999 | | 0.315093 | | | |
| 227 | 2 | | 0.074635 | | 0.9998 | | 0.07462 | | | |
| 228 | 3 | | 0.010475 | | 0.9997 | | 0.010472 | | | |
| 229 | 4 | | 0.000965 | | 0.9996 | | 0.000964 | | | |
| 230 | 5 | | 6.09E-05 | | 0.9995 | | 6.09E-05 | | | |
| 231 | 6 | | 2.67E-06 | | 0.9994 | | 2.67E-06 | | | |
| 232 | 7 | | 8.04E-08 | | 0.9993 | | 8.03E-08 | | | |
| 233 | 8 | | 1.59E-09 | | 0.9992 | | 1.59E-09 | | | |
| 234 | 9 | | 1.86E-11 | | 0.9991 | | 1.85E-11 | | | |
| 235 | 10 | | 9.77E-14 | | 0.999 | | 9.76E-14 | | | |
| 236 | | | | | | | | | | |
| 237 | Total | | 1 | | | | 0.99995 | 0.99995 | =(1-(J220*J221))^10 | |

Column C reports the result of applying the binomial probability distribution previously described for Branch Point 2 using $p_2$ = 0.05 (J220). Column E reports the result of applying the binomial probability distribution previously described for Branch Point 3 using $p_3$ = 0.0001 (J221). Column E is the probability that, given the k number of viremic cattle in column A, none (zero) is viremic upon arrival at the U.S. port of entry.

There is a problem with the calculation of joint probabilities when the simulation model is run. The problem results from applying the same total time from the onset of infection until arrival at the U.S. port of entry to all k animals. Using a different value for each animal is more appropriate. The result of not addressing this issue at this time will produce an exaggerated variation in the

overall results. However, refinement to address this limitation is not expected to affect the mean of the results significantly.

Column G reports the joint probability (column C x column E) that the number of cattle specified in column A is viremic upon entry to the pre-embarkation quarantine and that none of that number of cattle (zero) is viremic when arriving at the U.S. port of entry. We refer to the numbers in column G as the joint probabilities.

The sum of the joint probabilities for all 10 cattle (G237) equals the probability that none of the cattle exported per year from the affected region is viremic upon entry to the U.S. port of entry.

If APHIS applied this multiplicative approach directly to this risk assessment, Table 13 would consist of 30,000 rows rather than 10. However, an alternative is to use the formula, $(1-(p_2*p_3))^n$ [or (1-(J220*J221)) $^n$ as presented in H237]. This formula uses the $p_2$ value of Branch Point 2 (J220), as well as the $p_3$ value of Branch Point 3 (J221). The formula gives the same result as the multiplicative approach when n = 10. The observation that the values are the same with the two approaches supports the validity of the formula.